A NEW REPRESENTATION FOR SMALL-BODY GRAVITY ESTIMATION

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One of the options proposed in the 2009 Review of the U. S. Human Spaceflight Plans Committee calls for the consideration of robotic and human missions to asteroids.[1] Beginning in 2000, the NEAR Shoemaker spacecraft inserted into orbit about the asteroid 433 Eros, and studied it for a year before landing. The Hayabusa satellite journeyed to 25143 Itokawa, and operated in a proximity of the asteroid for several months before landing to collect a return sample. Additionally, the DAWN satellite, which was launched in 2007, will explore the dwarf-planet Ceres and the asteroid 4 Vesta in the next three years. These are just several examples of current interests in exploring asteroids.

For spherical bodies such as the Earth and Moon, the spherical harmonics gravity model is commonly used to approximate the gravity perturbations due to non-uniform mass distributions. As seen in Fig. 1, both the asteroids Eros and Itokawa are roughly elliptical in shape. For this reason, the elliptical harmonics model is more appropriate than the spherical one. As expected in the case of Eros, results indeed demonstrate the advantages of the elliptical harmonics gravity model.[2] For both the spherical and elliptical harmonics models, the partial sum diverges for points within the circumscribing sphere (or ellipsoid), thus limiting the minimum valid altitude. Unfortunately, not all asteroids are ellipsoidal and, thus, other gravity representations are required.

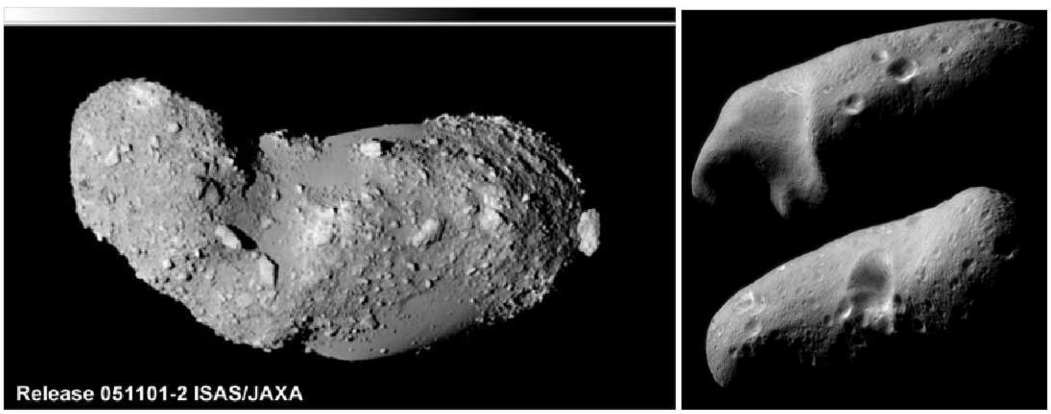

Figure 1: Asteroids 25143 Itokawa (left) and 433 Eros (right, as seen from opposite sides)

An alternative approximation models the asteroid as a constant density polyhedron.[3] Unfortunately, such a model is computationally expensive,[3] and is not easy to use in the orbit determination process.[4] The pubtree model, an interpolation model based on the polyhedron, improves computational efficiency while allowing evaluation at any point above the asteroid's surface.[5] Another proposed model uses a finite element representation of the asteroid, i.e. a collection of cubes or spheres with different masses, which may also be used for gravity field estimation.[4] However, this model may become computationally expensive and the estimation problem ill-conditioned if the number of masses is large.

In this paper, we explore an alternative for estimating the gravity field of a small body using observations of an orbiting satellite. Our approach uses two new mathematical tools, the quadratures (on a sphere) invariant under the icosahedral group, and a multiresolution representation of the gravity potential.[7] With the near optimal quadratures on the sphere, we minimize the number of parameters needed to recover a function. Also, instead of estimating the coefficients of the spherical harmonics, the new quadratures allow us to estimate directly the values of the gravity field since they offer an analogue of the Lagrange-type interpolation.

In the radial direction, our approach is based on a multiresolution representation of a solution of the external problem for the Laplace equation with boundary conditions on the sphere. Specifically, as derived in Section 5.3 of Beylkin and Monzón,[8] the gravity potential (U) may be approximated (to any finite but arbitrary precision) by

$$\tilde{U}(\rho,\phi,\theta)=\sum_{j\in\mathbb{Z}}e^{-(\log\rho)^2\sigma_j^2/2}Z_j(\phi,\theta) \tag{1}$$

where ρ is the ratio of the instantaneous orbit radius and the radius of the primary body ($\rho = r/R$), ϕ and θ are the latitude and longitude, respectively, and where

$$Z_j(\phi,\theta)=\frac{h}{\sigma_j\sqrt{2\pi}}\sum_{n=0}^{\infty}(n+1)e^{-(n+2)^2/(2\sigma_j^2)}U_n(\phi,\theta). \tag{2}$$

The parameter h appears as a result of discretizing an integral approximating $1/r^n$ for $n \geq 1$ in the usual solution of the Laplace's equation via the spherical harmonics, and $\sigma_j^2 = 2e^{jh}$. The choice of h determines the accuracy of the approximation. The functions $U_n(\phi,\theta)$ in Eq. 2 are the spherical harmonic part (of degree n) of the gravity potential in the usual solution of Laplace's equation via the spherical harmonics. However, in our approach we estimate functions $Z_j(\phi,\theta)$ directly, thus avoiding any use of the spherical harmonics. We note that, for a given accuracy, the exponential cutoff in Eq. 1 allows us to predict the order and degree of the subspace of spherical harmonics for the functions Z_j and, thus, choose the appropriate number of quadrature nodes for estimation. Using new quadratures, we need to estimate values of functions Z_j at these quadrature nodes. We also note that, as described in Beylkin and Monzón,[8] the number of terms in Eq. 1 depends only weakly on the required resolution and is relatively small. For a fixed ρ, only a few terms in Eq. 1 contribute to the potential.

In many ways, the resulting representation is similar to the cubed-sphere gravity model, which maps the sphere to a cube with bases defined on the cube surface for evaluation of the gravity field.[6] We choose the new representation of the potential (Eq. 1) since it is more economical for the purpose of estimation as it minimizes the number of measurements required.

With the mathematical approach now in place, this paper presents initial estimates of the gravity field for Eros given the NEAR mission as a baseline. Similar mission design and measurement accuracies will be used, with the NEAR15A gravity model[9] serving as the defined truth. Results presented include a comparison of the estimated gravity model, and differences in orbit propagation.

REFERENCES

[1] N. R. Augustine, W. M. Austin, C. Chyba, C. F. Kennel, B. I. Bejmuk, E. F. Crawley, L. L. Lyles, L. Chiao, J. Greason, and S. K. Ride, "Seeking a Human Spaceflight Program Worthy Of A Great Nation," tech. rep., U.S. Human Spaceflight Plans Committee, 2009.

[2] R. Garmier, J.-P. Barriot, A. S. Konopliv, and D. K. Yeomans, "Modeling of the Eros gravity field as an ellipsoidal harmonic expansion from the NEAR Doppler tracking data," *Geophysical Research Letters*, Vol. 29, No. 8, 2002, pp. 721–723.

[3] R. Werner and D. Scheeres, "Exterior Gravitation of a Polyhedron Derived and Compared with Harmonic and Mascon Gravitational Representations of Asteroid 4769 Castalia," *Celestial Mechanics and Dynamical Astronomy*, Vol. 65, 1997, pp. 313–344.

[4] R. S. Park, R. A. Werner, and S. Bhaskaran, "Estimating Small-Body Gravity Field from Shape Model and Navigation Data," *Journal of Guidance, Control, and Dynamics*, Vol. 33, January-February 2010, pp. 212–221.

[5] A. Colombi, A. N. Hirani, and B. F. Villac, "Structure Preserving Approximations of Conservative Forces for Application to Small Body Dynamics," *Journal of Guidance, Control, and Dynamics*, Vol. 32, November - December 2009, pp. 1847–1858.

[6] G. Beylkin and R. Cramer, "Toward Multiresolution Estimation and Efficient Representation of Gravitational Fields," *Celestial Mechanics and Dynamical Astronomy*, Vol. 84, 2002, pp. 87–104.

[7] C. Ahrens and G. Beylkin, "Rotationally Invariant Quadratures for the Sphere," *Proceedings of the Royal Society A*, Vol. 465, October 2009, pp. 3103–3125.

[8] G. Beylkin and L. Monzón, "Approximation by Exponential Sums Revisited," *Applied Computational Harmonic Analysis*, Accepted, doi:10.1016/j.acha.2009.08.011, 2009.

[9] A. S. Konopliv, J. K. Miller, W. M. Owen, D. K. Yeomans, and J. D. Giorgini, "A Global Solution for the Gravity Field, Rotation, Landmarks, and Ephemeris of Eros," *Icarus*, Vol. 160, 2002, pp. 289–299.